Computational Mathematics

Exercises Set: 2 (deadline Dec 27/2011)

1. Find Lagrange's polynomial for $y(x)=1+\sin(\pi\ x)$, $x\in(-1,1)$ using 5 points (x = -1, -0.5, 0, 0.5, and 1). Find the maximum relative error in (-1,1). Use numerical differentiation to compute $y'(x)$ on the same set of points with an accuracy of $O(h^2)$ and compute Hermite's polynomial. Make a graph of the results (the two polynomials and the original function).

2. Use Newton's forward differences formula to compute a 6^{th}-order polynomial approximation for the following set of points:

X	0.2	0.3	0.4	0.5	0.6	0.7	0.8
Y	0.185	0.106	0.093	0.24	0.579	0.561	0.468

Use 2-nd order accurate formulae to compute 1^{st} and 2^{nd} – order derivatives in each point. Find the Hermite polynomial and compute the differences in y(0.153) and y(0.658) for the two polynomials.

3. Compute the following integrals using (a) the *trapeze* rule, and (b) the *Simpson* rule with Romberg's improvement. In each case find the number of points needed to achieve accuracy of $O(10^{-6})$:

$$\int_1^2 \sqrt{x-0.1}\ dx \quad , \quad \int_0^1 \log(x+0.2)\ dx \quad , \quad \int_1^{1.4} \frac{1}{x-0.32}\ dx$$

4. Compute the following integrals, using the two methods suggested for each case. Compare the number of points needed to achive an accuracy of $O(10^{-8})$:

(a) $\int_0^{2\pi} e^{-x} \sin(10x)\ dx$ using the Simpson-Romberg rule and Filon's method

(b) $\int_0^{\pi/2} \log(x+1) dx$ using the Simpson-Romberg rule and Gauss's method.